Geostatistical Analysis of Libyan Clastic Thickness

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1. INTRODUCTION

The purpose of this case study is to illustrate some of the methods I outline in my book on geostatistics (Hohn, 1999). In particular, this paper outlines the calculation of declustered histograms and weights, the normal score transform, ordinary kriging of normal scores, backtransformation, sequential Gaussian simulation, and calculation of confidence envelopes from kriging variance and from a set of simulations. All geostatistical calculations are carried out with GSLIB routines (Deutsch and Journel, 1998); each section of this case study lists the specific routines used.

The data were obtained from a figure in a paper by Gumati and Kanes (1985), and comprise 38 measurements of the thickness of a Paleocene clastic unit in Libya. I originally selected this dataset because of the modest number of observations, and the relatively close spacing of observations relative to variation in thickness, yielding a simple variogram with small or negligible nugget effect. These characteristics of the dataset also make it useful for illustrating the basics of geostatistics, and providing a test set for students of geostatistics to test their understanding. Data may be downloaded from: http://www.wvgs.wvnet.edu/www/GeostatPetGeol.html#data

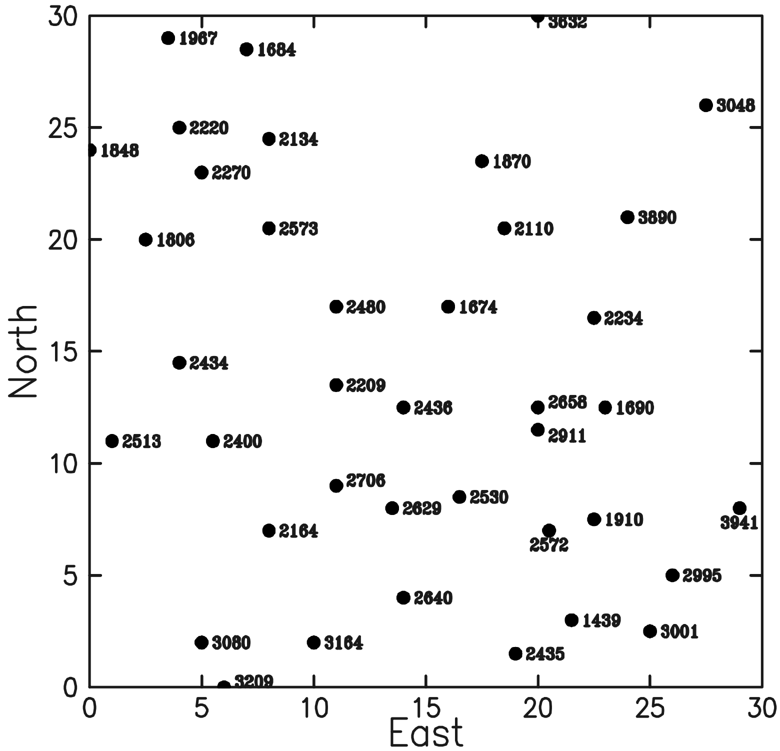

Figure 1. Thickness of a Paleocene clastic interval in Libya.

2. UNIVARIATE STATISTICS AND DECLUSTERING

Observed values of clastic thickness appear to be distributed evenly across the study area (Fig. 1). One must also consider whether observations are biased toward one part of the histogram of possible values. This occurs easily in development of oil and gas fields where naturally enough, a company is interested in wells with the largest potential production, and there is a bias toward drilling wells in areas considered most favorable, such as those with the thickest reservoir rock or highest average permeability. In a mineral survey, environmental assessment, or ecological sampling, one can set up the sampling pattern to avoid sampling parts of the distribution preferentially. In the petroleum business, previously-drilled wells comprise the “sample”, and locations of these wells are not selected to help the geostatistician; rather, the more successful the geologist or engineer in selecting well sites, the more the wells represent preferential sampling.

Geostatistical methods that use a transformation of data to a normal or near-normal distribution, or use the observed histogram, are going to be affected by bias in the observations. In this example, kriging of normal scores and sequential Gaussian simulation are to be used, both requiring a transformation of raw data to a normal distribution, followed by a back transform. Checking for bias is in order.

To do so, one can draw a map of the variable under analysis, and compare spatial patterns in the variable against sample location; ordinary kriging of the raw data should be adequate for this. If there is no bias, then there should be no clustering of data in high- or low-valued areas. This is a rather clumsy approach that depends a lot on judgment; it can be formalized by computing a declustered mean as described by Journel (1983). The study area is divided into square or rectangular nonoverlapping cells, the average computed for each cell from observations falling within it, and the results used to compute a grand average. The procedure is repeated for cells of different sizes, and results are plotted on a graph (Fig. 2).

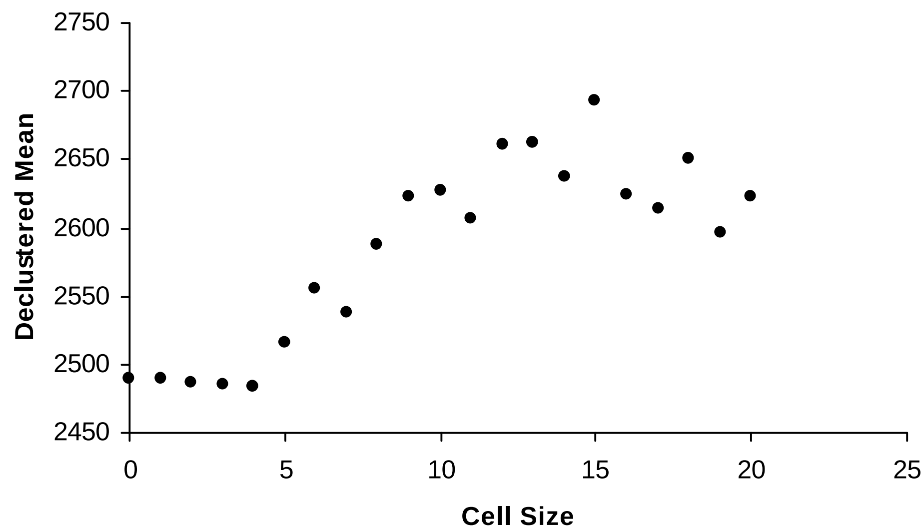

Figure 2. Relationship between grand mean of thickness averaged within cells and size of cells.

When observations are clustered in areas of high values, the graph exhibits a minimum for cell sizes larger than 0. In contrast, preferential sampling in low-valued regions results in a maximum in the graph, the case with the clastic thickness data. GSLIB procedure *declus* provides data for such a plot, and also declustering weights to be used while transforming to and from a normal distribution. In routines that compute frequencies, declustering weights are used to downweight samples in cells with a relatively large number of observations. Comparison of the histogram of unweighted thickness data (Fig. 3) and the weighted histogram (Fig. 4) shows that indeed, the lower tail of the latter has been downweighted.

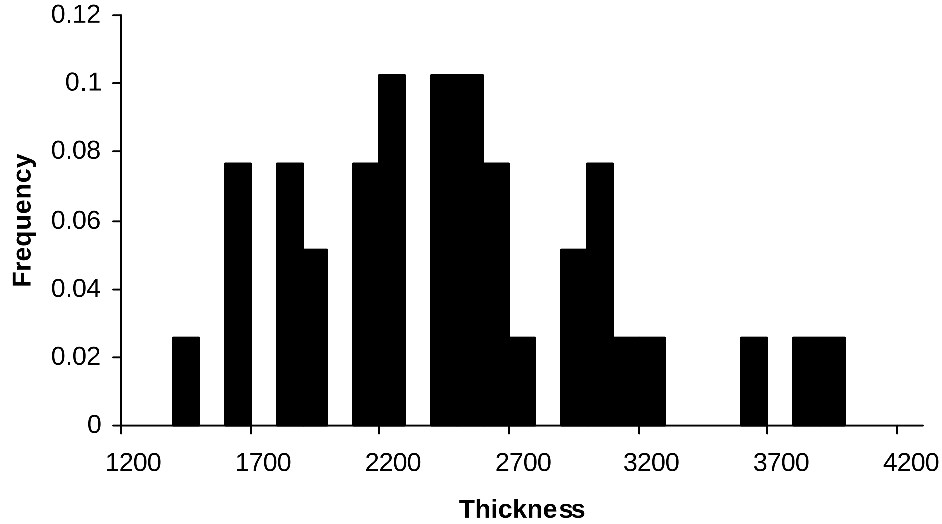

Figure 3. Histogram of clastic thickness.

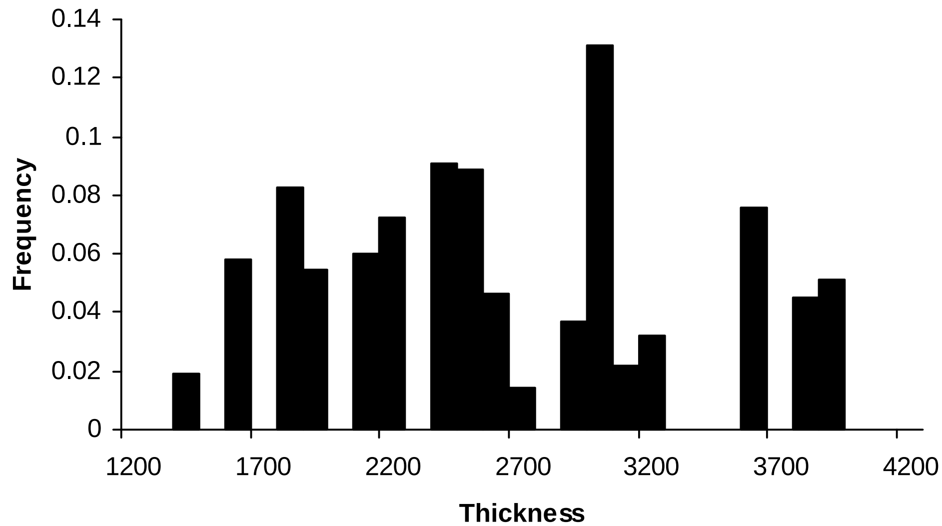

Figure 2. Histogram of declustered data.

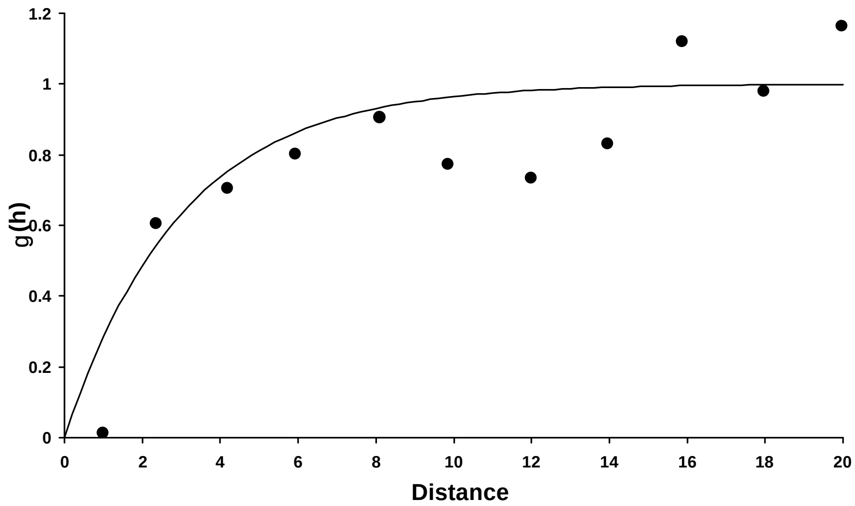

Figure 3. Variogram of normal scores of clastic thickness data.

3. VARIOGRAPHY

An omnidirectional variogram was calculated from normal scores of the observed thicknesses (Fig. 5). The number of observations was deemed too small for computing directional variograms with any degree of confidence. An exponential model was fitted with an effective range of 7.5, unit sill, and no nugget effect.

Normal scores and variogram values were computed with GSLIB routines *nscore* and *gamv*.

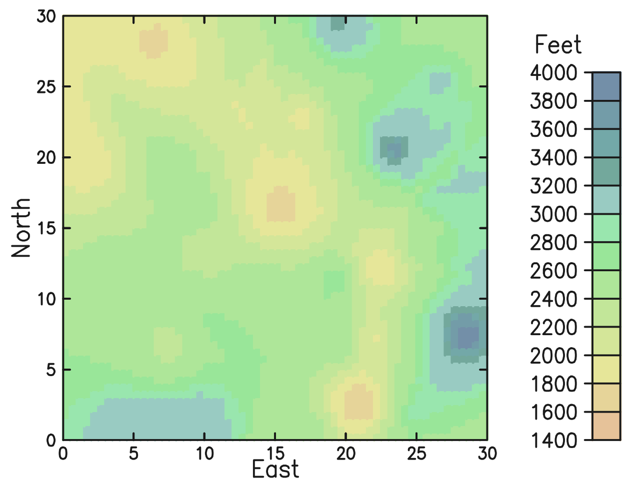

Figure 4. Kriged thickness of clastic Interval.

4. KRIGING

Normal scores of thickness and the variogram model were used in ordinary kriging to calculate estimates on a regular grid. At each grid location, confidence envelopes were determined by adding the estimation standard deviation to the estimate for the upper limit, and subtracting for the lower limit. Estimates and the upper and lower limits were then back-transformed to the original units of thickness (Figs. 6-8).

The resulting confidence interval for each estimate is analogous to that obtained by ordinary kriging of the raw data, but not quite the same. Because of the nonlinear transform of thickness data to the normal scores before the geostatistical operations, the confidence interval is not necessarily symmetric about the estimate in the original measurement unit. For instance, one would expect distributions with a long positive tail to yield an upper confidence limit farther from the estimate than the lower limit.

Ordinary kriging was performed with GSLIB routine *kt3d*, and backtransform with routine *backtr*.

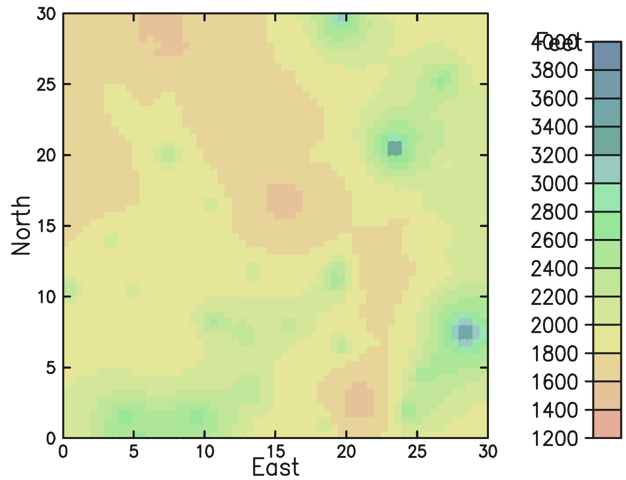

Figure 5. Lower "confidence limit" calculated by subtracting estimation standard deviation from kriged estimates of thickness.

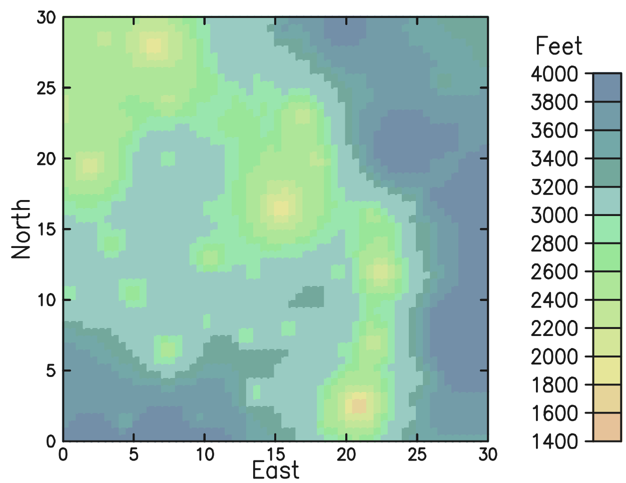

Figure 6. Upper "confidence limit" calculated by adding estimation standard deviation from kriged estimates of thickness.

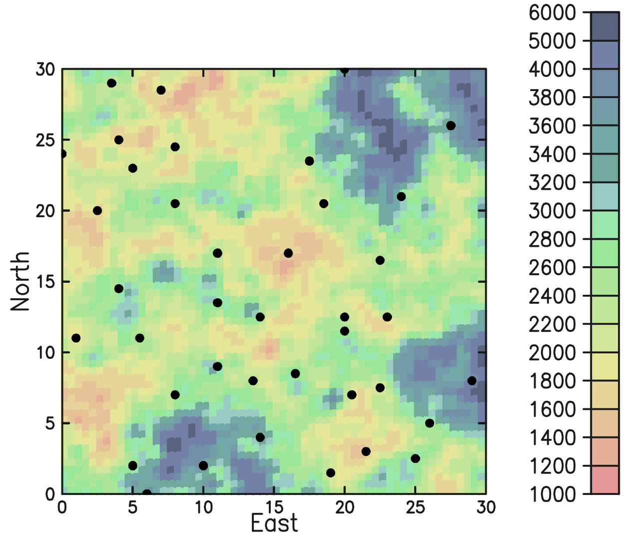

Figure 7. One simulation of clastic thickness.

5. SIMULATION

Sequential Gaussian simulation computes a value at each node that honors the variogram of normal scores and the transformed thickness data (Fig. 9). Simulated values are backtransformed to the original thickness units. Although only one simulation is shown, 100 were calculated to provide sufficient data for summary statistics.

For each grid location, the averaged simulated value should equal the kriged estimate because the same variogram and conditioning data are used, and ordinary kriging was selected in the simulation routine. Comparison of the map of kriged estimates (Fig. 6) and the average of the simulations (Fig. 10) shows this to be true for regions of thin clastics, less so in thicker areas such as in the northeastern corner. This could be related to how values are simulated in the upper tail of the distribution, or the relative lack of observations.

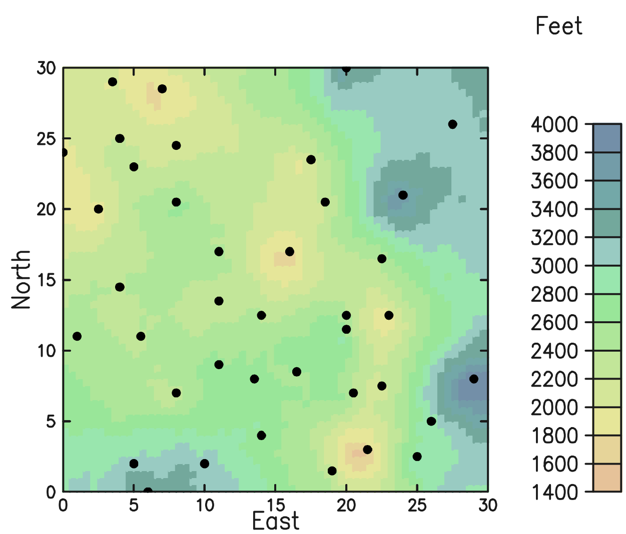

Fig. 10. Average of 100 simulations of thickness.

Confidence limits analogous to those from kriging can be obtained by calculating for each grid location the lower 16th and upper 84th percentile of simulated values for thickness (Figs. 11 and 12).

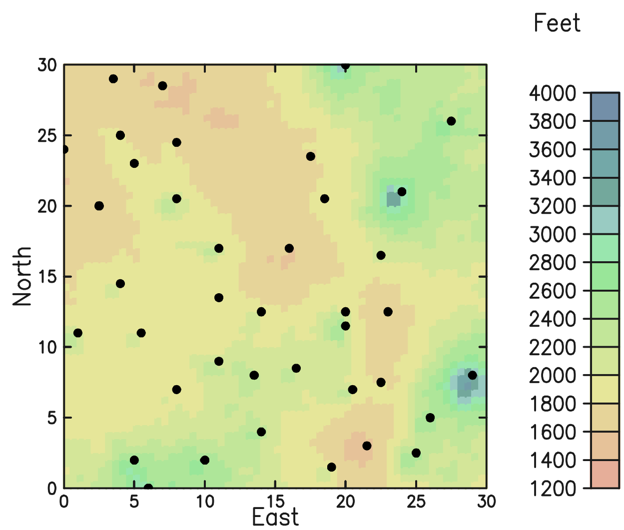

Fig. 11. Lower 16th percentile of thickness simulations.

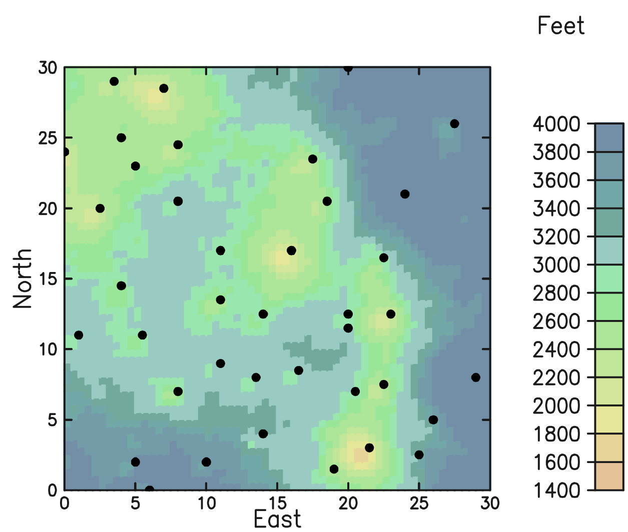

Fig. 12. Upper 84th percentile of thickness simulations.

6. REFERENCES

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